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Cristannus de Prachaticz's Treatises on the Astrolabe

Petr Hadrava
(Astronomical Institute, Academy of Sciences of the Czech Republic)

Alena Hadravová
(Centre for the History of Sciences and Humanities ICH, Academy of Sciences of the Czech Republic)

Abstract In the present contribution our computer methods of collation and statistical treatment of variant readings are summarized and illustrated on the example of the critical edition of the treatises on *Composition* and *Use of the Astrolabe* written by the 15th-century Czech scholar Cristannus de Prachaticz.

Keywords Computer-assisted Critical Editions. Digital Stemmatology. Treatises on Astrolabe. Cristannus De Prachaticz.

1 Introduction

The relations between medieval manuscripts are often complicated and the tradition that explains the origin of the texts may be misleading. A thorough study of textual tradition and in particular the preparation of critical editions may shed a better light on the genesis and the spreading of the treatises and consequently also on the interaction between the cultural centres where the manuscripts were written, copied or read. Such a work is difficult and laborious but, fortunately, it can be partly facilitated using the currently available computational technique.

An example of a topic which was intensively studied in medieval centres was the theory and practice of the construction and use of the astrolabe. Its description widely circulated in many medieval manuscripts. Treatises on *Composition* and *Use of the Astrolabe* were written by the Master of Prague University Cristannus de Prachaticz in 1407. The aim of our criti-

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cal edition of these treatises (Hadravová, Hadrava 2001) was to ascertain the original form of Cristannus's text and to investigate its relationship to its predecessors and followers. Our study of a great number of treatises on the astrolabe showed that Cristannus's formulation was highly original and successful, so that his treatises were widely spread throughout Europe. They even became the first texts on the astrolabe ever to be printed (Perugia 1477-1479), although they were then wrongly attributed to other authors such as Robertus Anglicus or Prosdocimo de Beldomandi.

For the preparation of the critical edition, which includes nine selected witnesses of the *Composition* (manuscripts, incunabula and early prints) and eighteen witnesses of the *Use of the Astrolabe*, we developed a method enabling a comparison amongst a large number of texts. The variant readings collected in the LaTeX source-file were then analyzed by a software which indicates statistically the relations between individual manuscripts and leads to suggest a *stemma codicum*. Our methods have been described and explained in Czech (Hadravová and Hadrava, 2001-2002). Although several similar methods have been developed since that time, our approach may still be applicable and the experience from its results useful for future work.

In the following we thus briefly summarize the contents of our edition and the circumstances of its preparation, i.e. the theory of astrolabe in Section 2, Cristannus's life and work in Section 3 and the spreading of his treatises on the astrolabe in Section 4. In Section 5 we present our method of collation and typesetting in LaTeX and compare it with the TEI-XML encoding. In Section 6 our statistical treatment of the variant readings is described and compared with some other computational methods in stemmatology.

2 The Astrolabe

The astrolabe is a universal astronomical and geodetic instrument (fig. 1). It was widely used from antiquity up to early modern times for observing the altitudes of celestial or terrestrial objects as well as for solving and demonstrating basic problems of spherical astronomy. Its principle is based on the stereographic projection, i.e. the projection of the sphere from its pole to the plane of equator.[1] The stereographic projection is advantageous for the construction of an astrolabe because its projection of

1 If the equatorial plane is treated as a plane of complex numbers r, the stereographic projection is given by equation $r=e^{i\alpha}\cos\delta\,/\,(1+\sin\delta)$, where α is the right ascension and δ is declination.

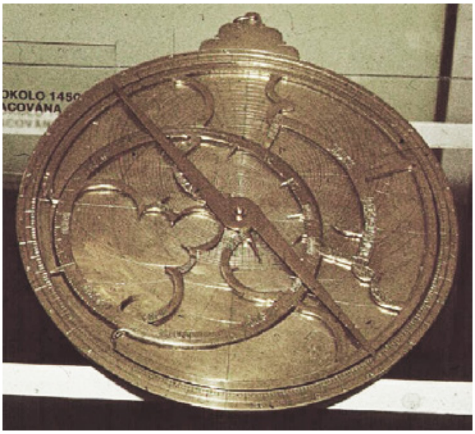


Figure 1. Astrolabe from about 1450. Prague, National Technical Museum, Inv. no. 2287

any circle on the sphere is a circle (or a straight line) in the plane.[2] A net depicting the ecliptic and the positions of selected bright stars rotates in the astrolabe on the background of a grid of horizontal coordinates, thus allowing one to find a correspondence between the sidereal time and the altitude of a star measurable through the alhidade and the angular scale on the instrument – cf., e.g., North (1974).

The astrolabe was described by Ptolemy (2nd century AD) in his Greek-language treatise *Planisphaerium*.[3] Ptolemy's treatise was translated into Arabic and adapted into several versions, which were later translated into Latin and heavily rewritten. More than forty different Latin treatises on astrolabe were used in the Middle Ages (Kunitzsch, 1982), one of the most popular of them being that by Pseudo-Massha'allah (2015).

Study of the astrolabe was a substantial part of the astronomy curriculum at universities in medieval Europe. In 1407, lectures on the astrolabe were read in the Prague university[4] by Cristannus de Prachaticz.

2 It can be seen from the equation $|r\text{-}2s/t|^2=|\sin\varphi\ (1+|s|^2)/t|^2$, where $t=1 - |s|^2 + \cos\varphi\ (1+|s|^2)$, for projection r of the points at angular distance φ from the centre of the circle on sphere which is projected to s. The stereographic projection also preserves angles and hence also shapes (but not sizes) of small figures. This is why it was also used for depiction of constellations and their mutual positions on whole hemispheres.

3 Ptolemy used the name 'astrolabe' for another instrument which he introduced in his *Almagest* for measurement of ecliptical coordinates. It is nowadays called 'armillary astrolabe' to distinguish it from the '(planisphaeric) astrolabe', named from Ptolemy's 'planisphaerium'.

4 Prague university was founded as the first one in the Central Europe by the Emperor Charles IV in 1348.

3 Cristannus de Prachaticz (Křišťan z Prachatic)

Cristannus was born after 1360 in the town of Prachatice in southern Bohemia. In 1388 he became Bachelor and in 1390 Master of Liberal Arts at Prague university, in which he spent rest of his life till his death in 1439. Cristannus was dean of the faculty of Arts in 1403-1404 and rector of the university in 1405, 1412-1413, 1434 and 1437. He dealt with mathematics, medicine, botany as well as theology,[5] but his fame is based nowadays on his astronomical work, within which his treatises on the astrolabe were the most important.

Cristannus was an older fellow, friend and supporter of the reformer Iohannes (Jan) Hus. It was on Cristannus's order that Jan Hus copied John Wycliffe's treatises which inspired Hus's criticism to contemporary Church. In 1415 Cristannus visited Hus in jail at the Council of Constance, where he was also imprisoned and released only thanks to intervention of the Emperor Sigismund. Cristannus was also greeted by Hus in his last letter before he was burned at the stake. Cristannus was one of the first two priests who started to offer the Holy Communion under Both Kinds in his parochial church in Prague, but he was a moderate Utraquist and was forced by radical Hussites to leave Prague for a while in 1420s. This religious and political orientation of Cristannus explains why he was 'persona non grata' for Catholic Europe, and his name disappeared from most copies of his treatises on the astrolabe, with his authorship later being entirely forgotten.[6]

Cristannus wrote two treatises on the astrolabe, namely the *Composition* (inc.: "Quamvis de astrolabii composicione tam modernorum quam veterum dicta habentur pulcherrima") and the *Use of the Astrolabe* (inc.: "Quia plurimi ob nimiam quandoque accurtacionem"). One can find in them some traces of treatises by Pseudo-Massha'allah which influenced also many other Latin texts. However, Cristannus's reprocessing of the topic is substantial and his work can be treated as an original one. His aim was to explain the subject more clearly than it had been done in the

5 Cristannus wrote several Latin treatises, such as *Algorismus prosaycus* and *Computus chirometralis*, *Tabula minucionum sanguinis et lunacionum* and *Collecta per magistrum Cristannum de Prachaticz de sanguinis minucione*, *Herbarius*; in Czech are written his works *Diverse Medicine* and *Medical Books* and others.

6 Cristannus is named as the author e.g. in mss [R], [L], [H], [O] (in the first two also the year 1407 of his lectures is given). A compromise selective approach was chosen by the scribe of ms. [K] saved in Hungarian Kalocsa who wrote: "Expliciunt utilitates astrolabii nove, satis valentes, Magistri Cristanni de Brachadicz, heretici perfidissimi pronunc, licet in composicione sive edicione earundem fuerit Cristianus" i.e. "Here ends a new and quite important treatise on the use of the astrolabe by Master *Cristannus* de Prachaticz, one of the worst heretics of the present day, although in the matter of writing and publishing the treatise he behaved as a Christian (*Cristianus*)" (cf. Hadravová, Hadrava 2001, 281).

other available treatises.[7] Cristannus's focus on didactic explanation was also characteristic for his mathematical treatises and it explains why his treatises on the astrolabe became very popular.[8]

4 Spreading of the Manuscripts and Early Prints with Cristannus's Treatises

The autograph of Cristannus's treatises on the astrolabe is not preserved. The oldest manuscript (our siglum [F] in the list of witnesses below) contains a note in the margin, dating it to 1408. It means that this copy was written immediately after the completion of the text. There are more than 80 known manuscript copies of the *Use of the Astrolabe* and 40 of the *Composition* which were written down to the mid-16th century. Moreover, Cristannus's treatises were also printed several times; we can find their texts in the well-known Perugia incunabulum of 1477-1479 (our siglum [u]), which was followed by other incunabula and early modern prints: Cologne 1478, Venice 1497-1498 (1494?, siglum [v]), Venice 1512, Venice 1521 (siglum [x]), Padua 1549. For our first critical edition of both treatises we have chosen the following nine texts of the *Composition*:

1. [C]: Prague, National Library, III C 2, fols 39*r*-42*v* (15th century)
2. [H]: Heiligenkreuz, Zisterzienserstift Bibliothek, Cod. 302, fols 121*r*-131*v* (1447)
3. [K]: Kalocsa, Fószékesegyházi Könyvtár (i.e. the Cathedral Library), 326, fols 10*r*-19*r* (after 1434)
4. [L]: Wien, Österreichische Nationalbibliothek, Cod. 5145, fols 66*r*a-71*r*b (15th century)
5. [M]: Wien, Österreichische Nationalbibliothek, Cod. 5184, fols 25*r*-36*r* (1482)
6. [O]: Wien, Österreichische Nationalbibliothek, Cod. 5228, fols 1*r*-14*v* (1500)
7. [R]: Rostock, Universitätsbibliothek, ms. math. phys. 4° 1[12], fols 173*v*-186*r* (1426)

7 This can be seen from his introduction to the *Composition*: "Quamvis de astrolabii composicione tam modernorum quam veterum dicta habentur pulcherrima, tamen, quia in eisdem quandoque sub paucis verbis magna latet sentencia, quam non nisi aliqualiter exercitati valent capere, igitur pro collectis tam valentis instrumenti utilitate quibusdam regulis conveniens erit pro complemento cepti operis planis tamen verbis composicionem eius conscribere, ut in unum hec collecta perfectum opus habeatur astrolabii" i.e.: "Although very nice words have been said about the construction of the astrolabe both in the old and in the modern times, nevertheless often in a few words there is hidden a great learning which can be comprehended only by the partly experienced. It will thus be convenient to write down in understandable words its construction as a complement to the started work, so as to collects some rules about the use of such a powerful instrument, and in this way to complete one work on the astrolabe which may be taken as perfect" (Hadravová, Hadrava 2001, 136).

8 For a more detailed description of Cristannus's life and work see Hadravová, Hadrava 2008 and 2001, 13-43.

8. [u] (incunabulum): *Roberti Anglici, viri astrologia prestantissimi, De astrolabio canones incipiunt*. Perugia, Petrus Petri, Johannes Conradi et Friedrich Ebert, 1477-1479 (ISTC ir00203000; copy: Milano, Biblioteca Trivulziana, Triv. Inc. C 127, 52-82)
9. [Y]: Florence, Biblioteca Laurenziana, ms. Laur. Ashb. 134 (208-140), 256a-283b (1419?)

and eighteen witnesses of the *Use of the Astrolabe*:

10. [A]: Oxford, Bodleian Library, MS. Canon. Misc. 436, fols 50*r*a-57*v*b (1468?)
11. [E]: Prague, National Library, V E 4b, fols 70*r*-85*r* (1479)
12. [F]: Prague, National Library, XIII F 25, fols 49*r*-68*r* (1407-1408)
13. [G]: Prague, National Library, IV G 10, fols 1*r*-19*r* (end of the 15th century)
14. [J]: Cracow, Biblioteka Jagiellońska, 3224, 459-537 (538-550 Additamenta), (1st half of the 16th century)
15. [K]: Kalocsa, Fószékesegyházi Könyvtár, 326, fols 52*r*-66*r* (after 1434)
16. [L]: Wien, Österreichische Nationalbibliothek, Cod. 5145, fols 58*r*a-66*r*a (15th century)
17. [M]: Wien, Österreichische Nationalbibliothek, Cod. 5184, fols 37*r*-49*v* (1482)
18. [N]: Wien, Österreichische Nationalbibliothek, Cod. 5210, fols 108*r*-132*r* (15th century)
19. [O]: Wien, Österreichische Nationalbibliothek, Cod. 5228, fols 15*r*-30*v* (1502)
20. [R]: Rostock, Universitätsbibliothek, ms. math. phys. 4° 1^{12}, fols 159*r*-173*r* (1426)
21. [S]: Berlin, Staatsbibliothek, Preussischer Kulturbesitz, ms. lat. oct. 438, fols 280*r*-291*v* (15th century)
22. [T]: Paris, Bibliothèque Nationale, Lat. 7282, fols 55*v*a-62*r*a (1468)
23. [u] (incunabulum): *Roberti Anglici, viri astrologia prestantissimi, De astrolabio canones incipiunt*. Perugia, Petrus Petri, Johannes Conradi et Friedrich Ebert, 1477-1479 (ISTC ir00203000; copy: Milano, Biblioteca Trivulziana, Triv. Inc. C 127, 1-51)
24. [v] (incunabulum): *Astrolabii quo primi mobilis motus deprehenduntur canones*. Venetiis, Paganinus de Paganinis, around 1497-1498 (1494?) (ISTC ia01171000; copy: Nelahozeves, Lobkowicz collection /formerly Prague NL/, Roudnice VII Ad 63)
25. [x] (early print): *Astrolabii quo primi mobilis motus deprehenduntur canones*. Venetiis, Petrus Liechtenstein, 1521. 4-0 (copy: Cracow BJ, Inc. 2696b)
26. [Y]: Florence, Biblioteca Laurenziana, ms. Laur. Ashb. 134 (208-140), 217a-255a (1419?)
27. [Z]: Genève, Bibliothèque Publique et Universitaire, 80, fols 1*r*-15*v* (15th century).

These manuscripts represent different versions of the text and their groups. Linguistic analysis of the textual variants has helped us to establish links between them, and the directions of their spreading from Prague. The details of this analysis exceed the aims and possibilities of the present contribution and they can be found in the edition (Hadravová, Hadrava 2001, 97-98 and 100-106). Before we deal with the statistical treatment, which confirmed the philological conclusions, we shall mention here only briefly some additional arguments.

In some manuscripts the place, year and name of the scribe are explicitly indicated.[9] An indirect evidence about the history of some copies can be gathered from Chapters 12 and 49 of the *Use of the Astrolabe*, where Cristannus mentions the time measured from the sunset by astronomical clocks "here in Bohemia" or "in Prague". Scribes abroad usually augmented these indications or directly replaced them by their own location (even if in their countries this 'old Czech time' was not used). We can thus find that the treatises were copied in the following regions and towns: *civitates Stagnales*, *partes Rheni*, *partes Alemanie*, *Saxonia*, *Cracovia*, *Polonia*, *Wienna*, *Ungaria*, *Italia*, *Roma*... Some of the manuscripts also contain tables of geographic coordinates of important cities. These tables help to trace back templates of the manuscripts because the scribes used to add the name of their own place on the last line (Hadravová, Hadrava 2001, 110-119).

The Italian branch of manuscript copies was a template for the aforementioned first incunabulum of the treatise on the astrolabe. Its editor, Ulyxes Lanciarinus Fanensis, pointed out in his foreword that this text is the best for students owing to its clarity, and he identifies it as "the newest rules by a foremost astrologer Robertus Anglicus". This name was used either for Robert of Chester,[10] who lived in the 12th century in England and Spain where he participated together with Hermann of Carinthia in the translation of Arabic texts, or for another Robert the Englishman, who commented in the 13th century Sacrobosco's treatise *De sphaera*. Regardless of the fact that Cristannus's text was really significantly more advanced than the texts from times of both these Roberts, neither of them had a reason to refer to *horologia in Italia*..., *in partibus Rheni et circa civitates Stagnales* and *in Praga*, which is what we can read in Lanciarinus's edition as well as in the subsequent prints. Antonio Favaro (1879) attributed the authorship of the Venetian print of 1521 to Prosdocimo de Beldomandi[11] because of its similarity to ms. Florence, Biblioteca Laurenziana, Ashb. 134 (our ms. [Y]), allegedly written by Prosdocimo in 1419, and because of a note penned in the outprint of the Jagellonian Library by Piotr Myszkowski – a scholar in Padua in 1530. However, Favaro and his followers did not investigate the other manuscripts with the text and their relations, nor did they try to explain the appearance of "Praga" (corrupted to "Parga" in ms. [Y]) in the text. The database "In Principio" (ver-

9 For instance ms. [R] was copied in Rostock 1426 by Conradus de Geysmaria, ms. [H] in 1447 by brother Ewald, professor in Heidelberg, ms. [E] by Master Iacob of Prague university.

10 Robert of Chester is named in ms. Wien, ÖNB 5311, fols 33*r*a-35*r*a, as a translator of another text on astrolabe classified by Kunitzsch (1982, 489-91) as a type RC.

11 Prosdocimo de Beldomandi (Padua, born between 1370-80, died 1428) is renowned especially through his treatise *Contrapunctus* (cf. e.g. Favaro 1879).

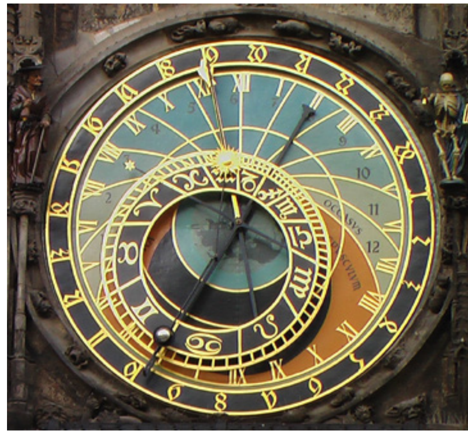


Figure 2. Astrolabic dial (with stereographic projection from the northern pole) of the Prague Astronomical Clock constructed by clockmaker Nicolaus of Kadaň

sion 2000) lists the text of *Composition* in ms. Madrid, El Escorial, Mon. de S. Lorenzo I II 7, fol. 163, which it attributes to the Alfonsinian scholar Aegidius de Tebaldis (Hadravová, Hadrava 2001, 96). Our study of this manuscript made already after the publication of the edition revealed that this copy is closest to the ms. [Y] and hence to the Italian branch of manuscripts and prints of Cristannus's *Composition*.

It is worth noting that Cristannus's text was also a basis for the treatises on the astrolabe written by the Viennese astronomer Iohannes von Gmunden in the 1420s or 1430s. Our comparative edition based on two manuscripts carrying his text (Hadravová, Hadrava 2001, 323-373) reveals that Iohannes von Gmunden included almost the whole of Cristannus's text in his treatise, but he added and further developed some parts, e.g. on terrestrial measurements.

5 Editions in LaTeX and TEI-XML

Our edition of Cristannus's treatises on the astrolabe was published in 2001, but we started to prepare it already in early 1990s. Regarding the relatively large number of witnesses to be included and the consequent size of the critical apparatus,[12] it became advantageous to prepare the edition as camera-ready, and the best option with the then available computers and software was to write it in LaTeX. The implementation of LaTeX distributed by CSTUG (i.e. Czech and Slovak TeX Users Group) included a handy and versatile editor program (CSED) allowing to deal with Czech diacritics. This

12 The *Composition* contains 1568 and the *Use of the Astrolabe* 3192 variant readings.

programme of editing is similar to KEDIT, and it proved to be useful for our method of collation of the individual texts – cf. figure 3; CSED allowed us to work with very long (potentially infinite) lines, of which the displayed part can be arbitrarily chosen using the cursor. We thus put a transcription of each manuscript or its greater part into a single line opened by its siglum, and we put these lines in a chosen sequence one below the other in one and the same window. By inserting spaces we shifted manually the rest of each line to the right so that the equivalent parts of the text formed a column in which the different variants were well visible. We then wrote the chosen reading into the resulting file of the critical edition created in another window at the bottom,[13] and we also indicated its variants in LaTeX-footnotes.

In the above described procedure we generated a LaTeX source-file which in fact anticipated the syntax developed later[14] by the Text Encoding Initiative (TEI) for encoding the critical editions (cf. the on-line document TEI P5). The similarity of both styles of encoding of the critical edition can be seen in the following example of the text:

> Quamvis de astrolabii[1] composicione tam modernorum quam veterum dicta[2] habentur[3] pulcherrima[4]
>
> [1] astrolabii composicione α : composicione astrolabii **C**   [2] dicta α : dictis **M**   [3] habentur α : habeantur **Ou**   [4] pulcherrima α : plurima **M**

taken from our edition of the Cristannus's *Composition of the astrolabe*. In LaTeX it has the form:

```
Quamvis de astrolabii\footnote{ astrolabii composicione
{\bf {\qa}KLMO}: composicione astrolabii {\bf C}}
composicione tam modernorum quam veterum dicta\footnote
{ dicta {\bf {\qa}CKLO}: dictis {\bf M}}
habentur\footnote{ habentur {\bf {\qa}CKLM}: habeantur {\bf Ou}}
pulcherrima\footnote{ pulcherrima {\bf {\qa}CKLO}: plurima
{\bf M}}
```

which can be transcribed into XML in the TEI convention as:

```
Quamvis de <app><lem wit="#H #K #L #M #O #R
#u #Y">astrolabii composicione</lem> <rdg
wit="#C">composicione
```

13 Actually, we modified only the text of the basic manuscript copied initially into this window, because its text turned out to be often the best one.

14 The first version of TEI was released by the end of the year 2007, i.e. more than a decade after our approach was first developed.

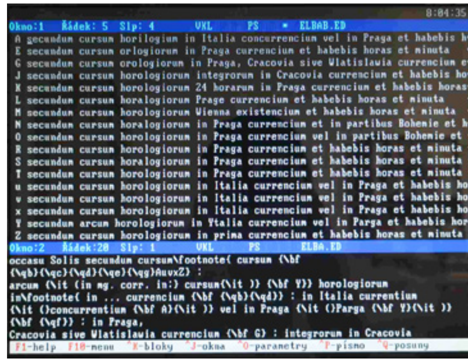


Figure 3. A screenshot of CSED with the edition of Cristannus's Use of the astrolabe (chapter 49) during its preparation

```
astrolabii</rdg></app>
tam modernorum quam veterum <app><lem wit="#C #H #K #L #O
#R #u #Y">dicta</lem> <rdg wit="#M">dictis</rdg></app>
<app><lem wit="#C #H #K #L #M #R #Y">habentur</lem>
<rdg wit="#O #u">habeantur</rdg></app>
<app><lem wit="#C #H #K #L #O #R #u #Y">pulcherrima</lem>
<rdg wit="#M">plurima</rdg></app>
```

The main difference between these two methods of encoding is that the preferred text is to be written in LaTeX both in the base text as well as in the apparatus footnote (where it can be abbreviated to the first and the last word of longer passages), while in TEI it is denoted as a 'lemma' inserted from the apparatus into the base text (cf. TEI P5, Chapter 12.1.2). For the rest, these styles can be converted from one to the other practically by a straightforward interchange of the brackets with keywords in LaTeX to the corresponding XML elements in TEI.[15]

The equivalence of both methods in their logical structure is a natural consequence of the nature of the task of the critical edition. It also results from this equivalence, that analogous procedures of preparation of the editions as well as further processing of the texts like the statistical treatment described in the next Section can be used for both methods.

15 In our example the keyword \qa stands for α, which is an abbreviation of the group of manuscripts HRuY, which often share identical variant readings.

6 Statistics of Variant Readings and Stemma Codicum

The critical apparatus yields evidence about the relationship between the manuscript texts, and it is thus self-suggesting that a mathematical treatment of variant readings may be helpful in the determination of a *stemma codicum*. This possibility has been studied a long time before computers started to be used for the typesetting of the editions (e.g. by Quentin 1926, Greg 1927), but it became much more easily applicable on electronically encoded texts. Plenty of statistical methods have been designed for stemmatology or modified from similar methods in other disciplines (cf. e.g. Baret, Macé and Robinson 2006, Roos and Heikkilä 2009) like the determination of phylogenetic trees in evolutionary biology (cf. e.g. Felsenstein 2004). Hereby, we shall summarize our own statistical method of ‘binary correlations’, developed and applied to examine the textual tradition of Cristannus’s treatises on the astrolabe as documented by the aforementioned LaTeX-edition, and we shall compare it with the results yielded for the same data by some of the recent methods.

Let us define the binary correlation of two particular witnesses as their agreement in a variant reading. The rigid syntax used in the LaTeX-encoding of the critical apparatus enabled us to debug a Fortran code which distinguishes in the source-file of the edition each group of witnesses with an identical reading. It is thus possible to count how many times each pair of witnesses appears in the same group. For n witnesses we thus get $n(n\text{-}1)/2$ independent counts of the binary correlations. In the following table we give as an example the values of correlations in Cristannus’s *Composition* (cf. Hadravová, Hadrava, 99; for the more extended table for the *Use of the Astrolabe* see 107-108).

	H	R	u	Y	M	L	O	C	K
H		1129	1112	1086	1019	1015	773	740	467
R	1129		960	956	925	903	718	681	440
u	1112	960		1288	883	883	764	662	422
Y	1086	956	1288		870	882	770	660	415
M	1019	925	883	870		819	647	608	390
L	1015	903	883	882	819		666	614	388
O	773	718	764	770	647	666		491	358
C	740	681	662	660	608	614	491		387
K	467	440	422	415	390	388	358	387	

A higher number of correlations indicates a closer similarity between the two witnesses and thus also their possible dependence.

It would be possible to draw up a tree in the sense of the graph theory,[16] maximizing the sum of weights of the edges given by the values of the binary correlations. Such a solution would be analogous to the 'minimal evolution tree' in biology, because a high correlation of two witnesses means a low ('evolutionary') change between them, so that the distance between two witnesses, which is minimized in phylogenetics, is the difference between the total number of witnesses and the correlation of this pair of witnesses. The mathematical problem of selecting the optimal tree has been solved by Jarník (1930).[17] We can see in figure 4, depicting such an optimal tree for Cristannus's *Composition* and *Use of the Astrolabe*, that this method confirms our choice of the basic manuscripts [H] and [F], respectively. However, such a straightforward algorithmic solution ignores the possibly useful information represented, e.g., by the dating of individual witnesses or other facts. It may thus lead to paradoxical conclusions such as a younger manuscript joining distinct families of older manuscripts. In fact, we can find such a violation of causality (chronology) between the witnesses [u] and [Y] in Cristannus's *Composition*, and between witnesses [J] and [G], [u] and [A] and [x] and [v] of the *Use of the Astrolabe*.

It is necessary in such cases to assume the existence of some hypothetic common template of the two families which was later faithfully copied into the preserved witness. The existence of common (unpreserved) templates is assumed in phylogenetics where the binary trees are searched for. It means that every branch splits into two edges only at each additional hypothetical vertex. The structure of the tree and the weights of its edges must be reconstructed so as to give the distances between the real vertices (the witnesses) as a sum of all edges on the path joining them. This mathematical problem generally does not have an exact solution,[18] however, the solution can be approximated by various methods which may involve

16 The tree is an undirected graph consisting of the vertices (represented by the individual witnesses) joined with edges (i.e. lines between the witnesses), such that there is exactly one path connecting any two vertices. A real *stemma* need not be a simple tree if some witness is influenced by more than one template and hence the resulting graph is cyclic (i.e. it contains a closed path). There can be found n^{n-2} different unrooted trees between n vertices.

17 Jarník's algorithm chooses in the first step the optimal edge (i.e. with the smallest distance and highest correlation) and then in subsequent steps always the optimal edge which joins any of the already connected vertices with some of the remaining vertices. This algorithm was later several times rediscovered e.g. by Robert Prim in 1957.

18 For n given tips (the preserved witnesses) there must be added in such a tree n-1 hypothetical vertices (one root and n-2 bifurcations) connected with altogether 2n-2 edges. The total number n (n -1)/2 of distances is thus for n>3 higher than the degree of freedom of the solution given by the number of weights of the edges (the difference between the edges from the root does not influence the distances between the tips). Such a rooted bifurcation tree can be constructed in $(2n-3)!/2^{n-2}/(n-2)!$ topologically different ways.

additional conditions such as optimizing the sum of all edges (i.e. minimizing the number of evolutionary changes). One of these methods is the so called Neighbour-joining (cf. Felsenstein 2004, 167). This method gives in our case of Cristannus's treatises the structure of the trees shown in figures 5 and 6.

We can see from these diagrams that the method of Neighbour-joining really indicates closer relations between witnesses precisely where we expect them, e.g. between the oldest manuscripts [F], [R], [H] or within the Italian branch including [Y] and the early prints. However, it appears very unlikely that the manuscript [F] of the *Use of the Astrolabe* should have been preceded, within one year, by six generations of templates from which most of the variants should have developed. Similarly, the basic manuscript [H] and its close relative [R] of the *Composition* should be preceded by four generations, but while they have diversified from [Y] in the third generation only, in the *Use of the Astrolabe* this diversification should have taken place already in the first generation. These problems are a consequence of the fact that the assumptions of these algorithmic methods are – compared to evolutionary biology – much less acceptable for the textual tradition of manuscripts and, on the other hand, the additional information available for manuscripts is ignored in them.

The approach we chose in the edition of Cristannus's treatises was thus different. Since there is no preserved autograph, we can suppose that the witness with a high number of correlations to all others is the closest to it. If we plot the positions of all witnesses into a graph in which the time of their origin is running down on the ordinate and the binary correlation with the basic witness divided by the total number of variant readings containing this witness decreases to the right on the abscissa, then the causally connected witnesses indicated by their high binary correlation should lead right down from the template to its copy (unless the copy corrects obvious mistakes of the template). A practical example from our edition can be seen in figure 7; this graph does not pretend to give a final *stemma*, but it suggests possible relations between the witnesses, which should be verified or disproved by a more detailed philological analysis.

It should be kept in mind that the numerical values obtained by our method described above, or by any other statistical treatment of the critical apparatus, are dependent on the editor's choice concerning which variants are worth to be included. To get some promising result, the treatment should be as homogeneous as possible already in the stage of collation of the witnesses. Some normalization may be needed in the case of extended omissions. However, even in an ideal case the statistics can be only a supporting criterion. In an analogy with the sentence "astra inclinant, non necessitant", we could state that statistics can indicate or suggest the *stemma* but cannot to determine or prove it.

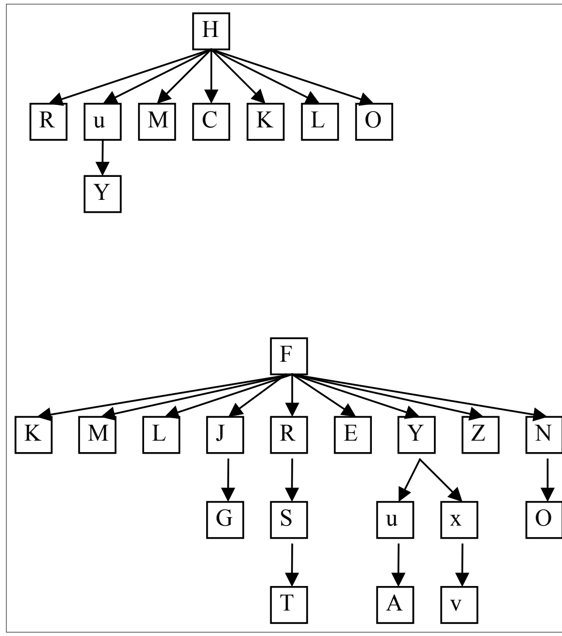


Figure 4. Optimal trees joining the witnesses of Cristannus's Composition (up) and Use (bottom)

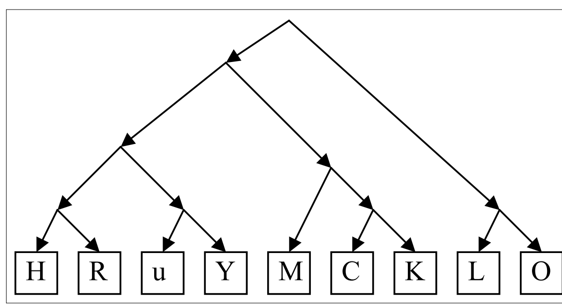


Figure 5. Neighbour-joining tree for Cristannus's Composition of the Astrolabe

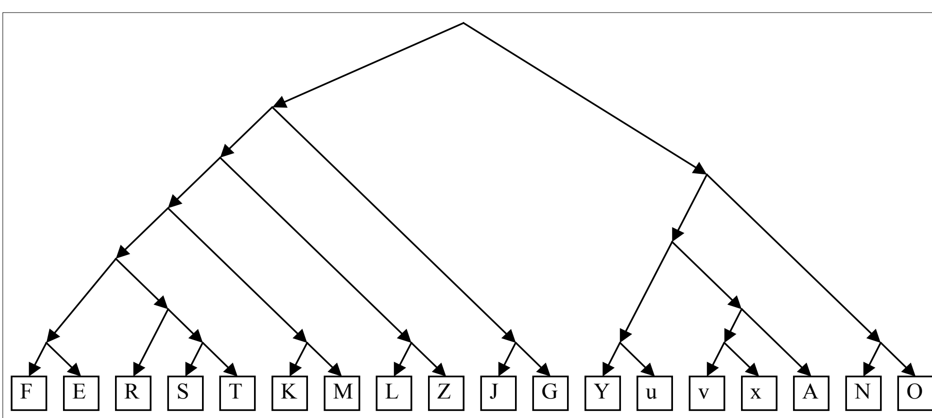
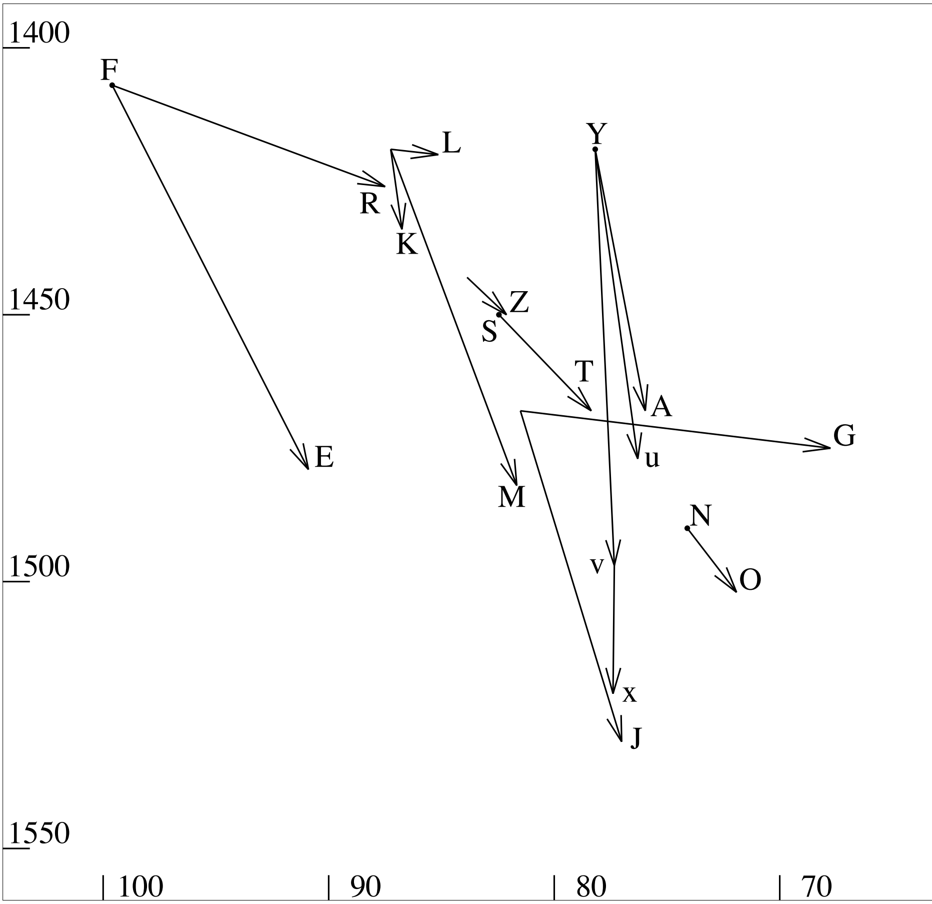


Figure 6. Neighbour-joining tree for Cristannus's Use of the Astrolabe



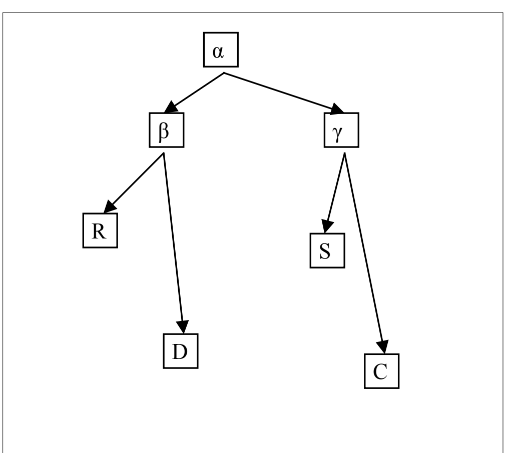


Figure 7. Graph of statistically found relations between witnesses of Cristannus's Use of the Astrolabe

Figure 8. Neighbour-joining tree for a subset of four witnesses

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Appendix

An interesting simple example of limitations of the standard algorithmic methods in stemmatology can be seen in the case of another text (Hadravová 2017). The four oldest preserved manuscripts of this text are [R] from the 12th century, [S] from the break of the 12th and 13th century, [D] from the break of the 14th and 15th century and [C] from 1401. The manuscripts differ in terms of the number of of individual chapters in which the text is organized. These four witnesses contain altogether 231 chapters out of the total number 275 found also in other younger manuscripts. If we count the correlations in presence of the chapters we find values for individual pairs: $c_{SC} = 270$, $c_{RD} = 260$, $c_{SD} = 253$, $c_{DC} = 252$, $c_{RS} = 242$ and $c_{RC} = 241$. Their complements to the total number 275 give distances of these vertices $d_{SC} = 5$, $d_{RD} = 15$, $d_{SD} = 22$, $d_{DC} = 23$, $d_{RS} = 33$ and $d_{RC} = 34$. A straightforward solution by Neighbour-joining algorithm yields the tree shown in figure 8, which is consistent with the dating of the manuscripts (depicted by the vertical displacement of the individual items in fig. 8). This solution also gives the distances from the hypothetical vertices $d_{\beta R} = 13$, $d_{\beta D} = 2$, $d_{\beta\gamma} = 18$, $d_{\gamma S} = 2$ and $d_{\gamma C} = 3$ which precisely reproduce all distances between the tips (e.g. $d_{RS} = d_{\beta R} + d_{\beta\gamma} + d_{\gamma S} = 13 + 18 + 2 = 33$). However, if we inspect in detail how many and which chapters are present in each witness, we find that [D] contains 230 chapters from which 15 are missing in [R] (which has 215 chapters), namely the chapters 170-174, 224, 228-229, 245-248 and 270. It means that the true $d_{\beta R} = 15$ instead of 13 and $d_{\beta D} = 0$ instead of 2. The only chapter, which is missing in [D] (and also in [R]) but is contained in [S] and [C], is chapter 225. It thus follows from the assumption of 'minimal evolution' that [β] contained the same 230 chapters as [D], from which 15 were lost by [R], [α] contained 231 chapters (i.e. also no. 225 lost by [β]). We can find that [S] contains 210 chapters including nos. 49, 107 and 205 which are missing in [C]. Vice versa, [C] contains amongst its 209 chapters also nos. 34 and 259 missing in [S]. Consequently, [γ] should have 212 chapters, i.e. it lost 19 chapters from [α] which we can identify as nos. 170–176, 224, 228-229, 245-248 and 270. This means that the true distances (number of changes in copies) are $d_{\alpha\beta} = 1$ and $d_{\alpha\gamma} = 19$, which cannot be determined by the Neighbour-joining and which give together distance $d_{\beta\gamma} = 20$ instead of 18. The failure of the method of Neighbour-joining is a consequence of the violation of the assumption that the distances between the tips are always the sum of the positive lengths of the edges (branches) on the path joining them. In our case $d_{R\gamma} \neq d_{\alpha\gamma} + d_{\alpha\beta} + d_{\beta R}$, because in the first and the third branch the same chapter 224 was omited and the result of these two independent but identical changes decreases instead of increasing the distance of [R] from the family [S] + [D] developed from [γ]. Although such a coincidence is more likely for omissions (which can take place also on a detailed level of variant readings) similar independent identical evolution cannot be excluded also in other cases and this limits the reliability not only of Neighbour-joining but of all algorithmic methods.

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